Introduction to Computer Graphics

Marie-Paule Cani & Estelle Duveau

04/02 Introduction & projective rendering
11/02 Prodedural modeling, Interactive modeling with parametric surfaces

25/02 Introduction to OpenGL + lab: first steps & modeling

04/03 Implicit surfaces 1 + lecture/lab: transformations & hierarchies

11/03 Implicit surfaces 2 + Lights & materials in OpenGL
18/03 Textures, aliasing + Lab: Lights & materials in OpenGL
25/03 Textures in OpenGL: lecture + lab
01/04 Procedural & kinematic animation + lab: procedural anim
08/04 Physics: particle systems + lab: physics 1
22/04 Physics: collisions, control + lab: physics 2
29/04 Animating complex objects + Realistic rendering

06/05 **Talks: results of cases studies**

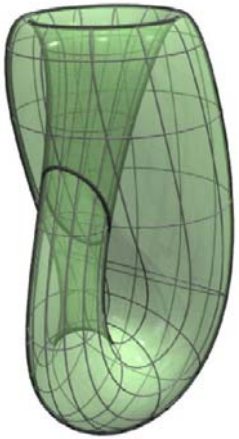
Drawbacks of Boundary Representations

- Complex shapes with splines ?
  - Branches ?
  - Arbitrary topological genius ?

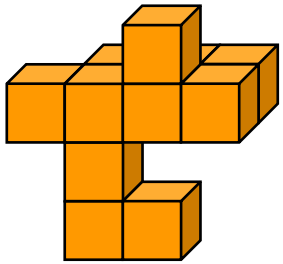
  Partly solved by subdivision surfaces

- Surrounding a volume?
  - Avoid Klein bottles!
  - Prevent self-intersections

  Make them impossible?

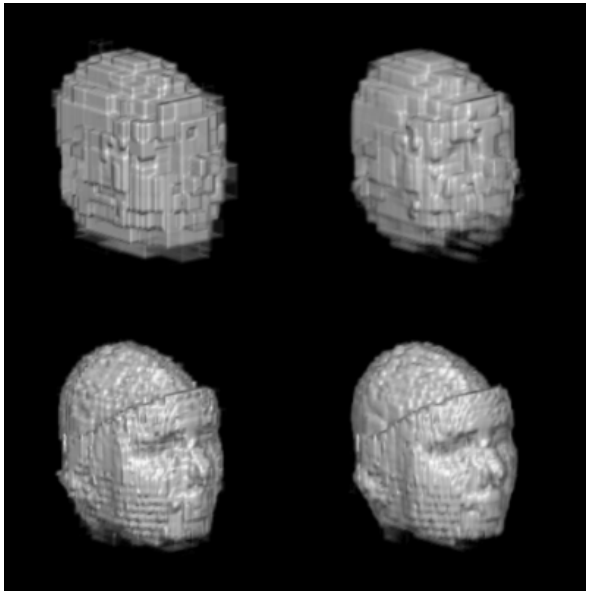


Solution
Smooth Volume Representation



Discrete volume

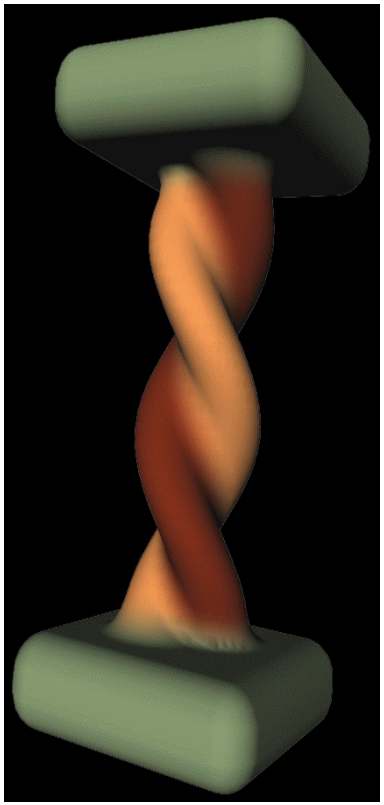
Voxels



Smooth volume

Remains smooth when we zoom in

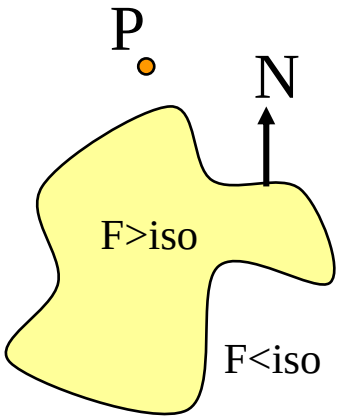
Can be converted to a mesh at any scale



Implicit surfaces

Defined by an *Implicit Equation*

$$S = \{ P(x,y,z) \ / \ f(x,y,z) = iso \}$$

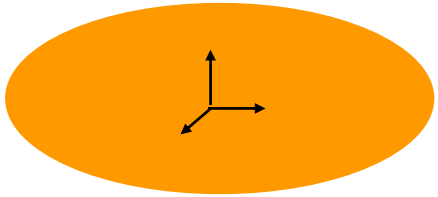


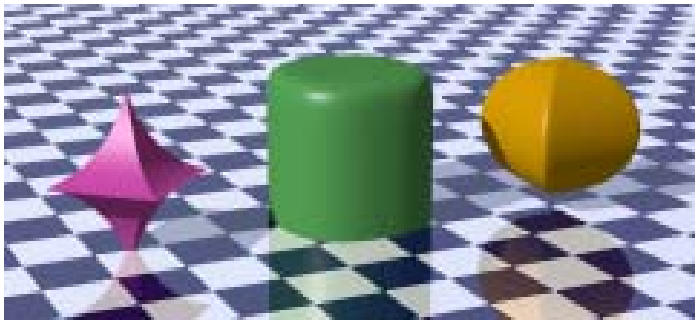
- $(f: R^3 \rightarrow R)$ is the «field function»
- Surface normal : $N = - \nabla f$
- Characterizes a volume! $f(x,y,z) > iso$
  - « in/out » test (used for collisions, ray tracing…)
- Smoothness: *S* and *f* have same degree of continuity

History: Solid Geometry

Volumetric primitives $S = \{ P(x,y,z) \ / \ f(x,y,z) = iso \}$

- Spheres, ellipsoids $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
- Cylinders, cones …
- Super-ellipsoids $\frac{x^n}{a^n} + \frac{y^n}{b^n} + \frac{z^n}{c^n} = 1$

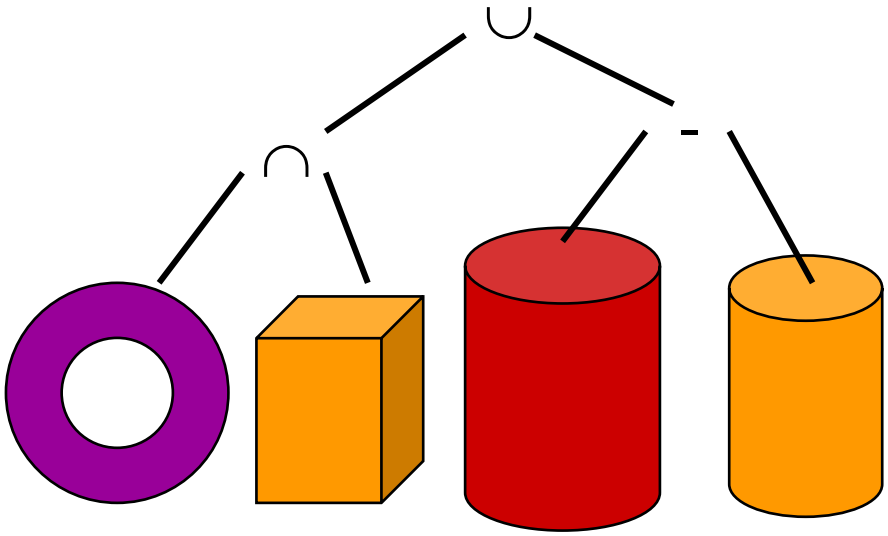




Constructive Solid Geometry

Developed for Computer Aided Geometric Design **(CAGD)**

- Solid primitives
- Boolean operators
  - Union (or)
  - Intersection (and)
  - Difference (not)
- Construction tree



Describes the history of construction in a compact, intuitive way

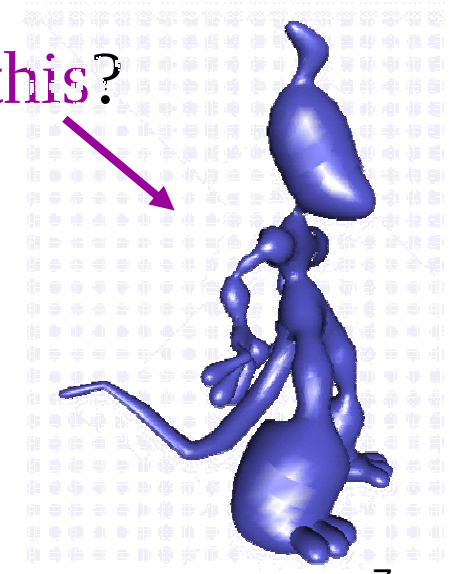
Problem: limited shapes

Free form primitives ?

$$S = \{ P(x,y,z) \ / \ f(x,y,z) = iso \}$$

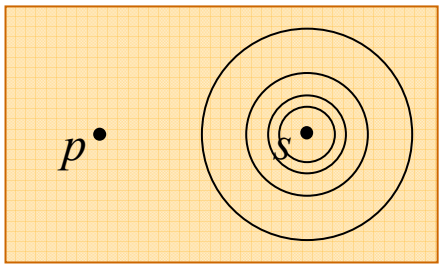
f polynomial (algebric surface), or other smooth function

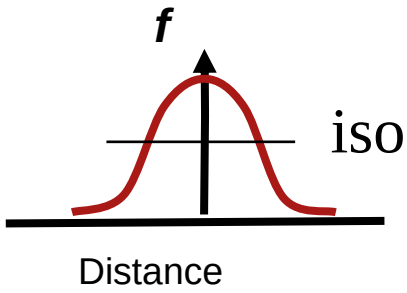
- What should the equation of f be to model this?
- How can a user control an implicit shape?
  - Intuitive control
  - Locality
  - Allow deformations



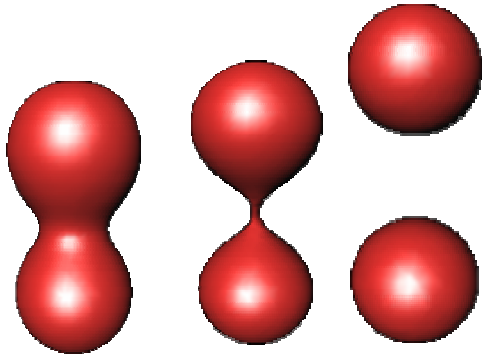
Idea (1982)
Blinn Objects "Blobs"

- Primitive generated by points *S*
  - *f* decreasing function of the distance



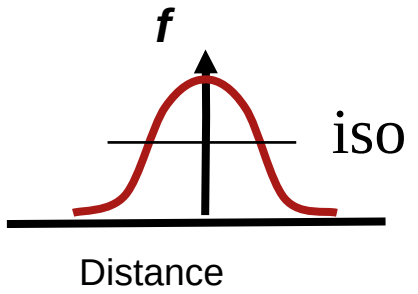


- Union : $f = max\ (f_1, f_2)$
- Intersection : $f = min\ (f_1, f_2)$
- **Blending** : $\boldsymbol{f = f_1 + f_2}$



Idea (1982)
Blinn Objects "Blobs"

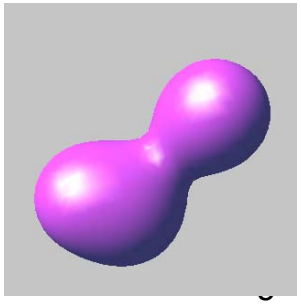
- Exponential field $f_i = e^{-\frac{d(P-Si)^2}{2}}$



  - \+ Very smooth
  - – No local control
  - – Everything is to be recomputed if a point moves

- Extension to blend primitives of different sizes

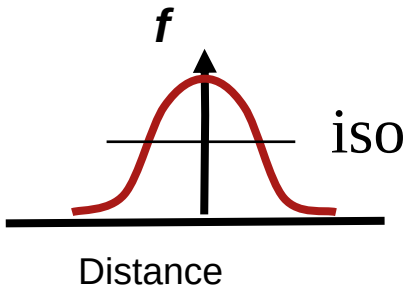
$$f_i = k_i e^{-\frac{d(P-S_i)^2}{R(S_i)^2}}$$



Make implicit surfaces local?
(1985-1990)

Field function with compact support!

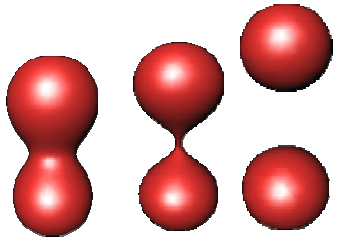
- piece-wise polynomial functions in $d(P,S_i)^2$



- Metaballs *[Nishimura 1985]*
  - *if* $0<d<1/3 \quad f_i = 1- 3\ d^2$
  - *if* $1/3<d<1 \quad f_i = 3/2\ (1-d^2)$

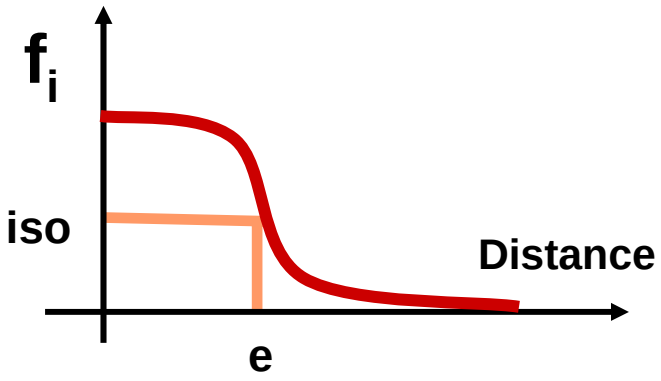
- Soft Objects *[Wyvill MP W 1986]*

  - if $0<d<1 \qquad f_i = -4/9\ d^6 + 17/9\ d^4 - 22/9\ d^2 + 1$

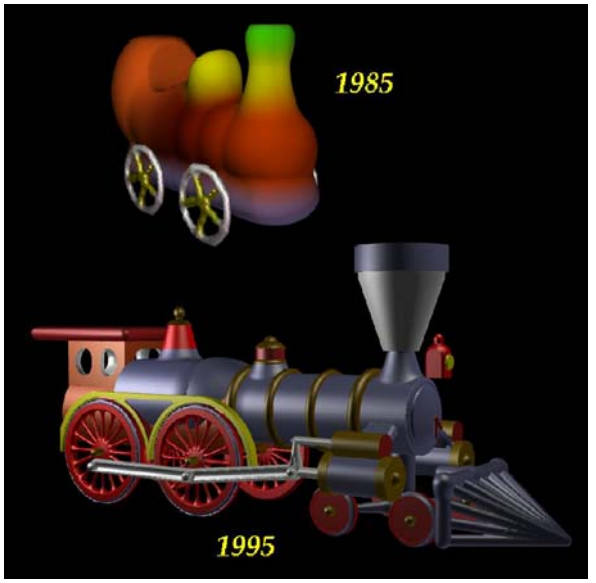


Choice of the field function?

- e gives the thickness of an isolated primitive
- The slope affects the final shape!
- Using $(-f_i)$ instead of f_i carves the shape
  - need of a flat tangent in zero



B. Wyvill 85-95

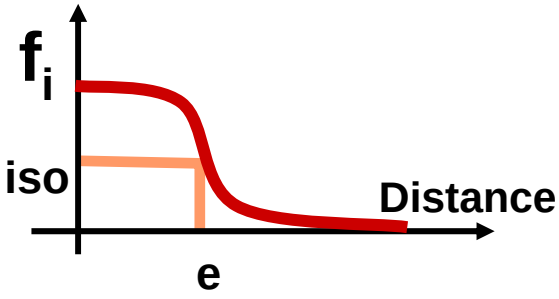


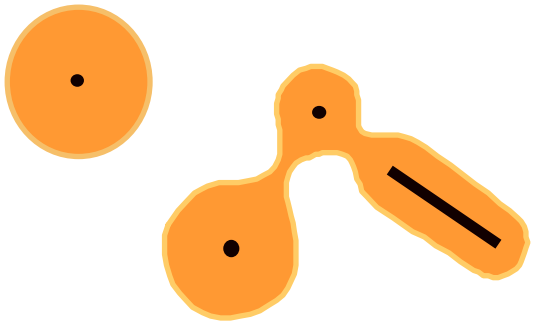
Extensions (1990-1995)

Skeleton-based Implicit Surfaces

Idea: Use any primitive S_i as a skeleton

- $S = \{ P / \sum f_i(P) = iso \}$
- f_i decreasing function of $d(P,S_i)$



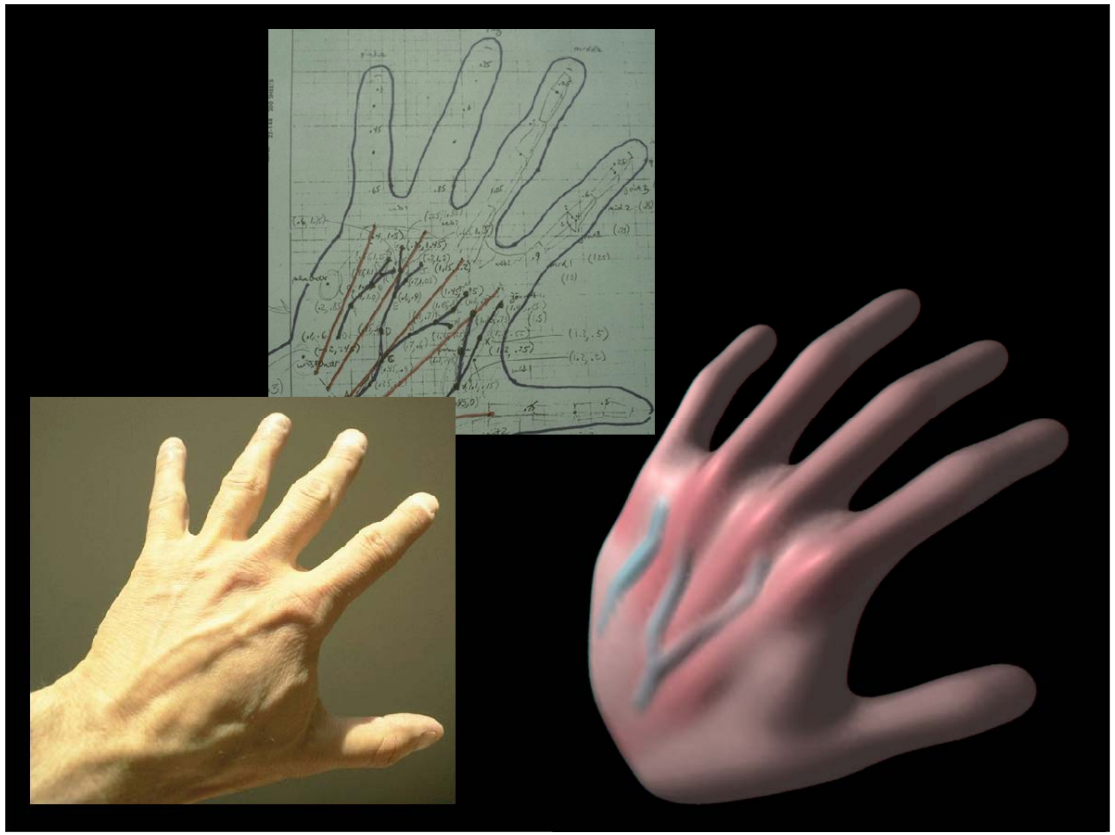


Point, segments, disc, cylinder

- Intuitive control, deformation, change of topology

Extensions (1990-1995)

Skeleton-based Implicit Surfaces

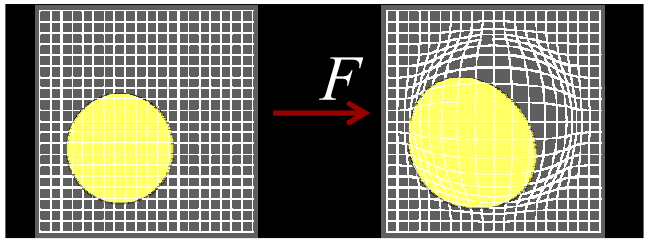


J. Bloomenthal
1995

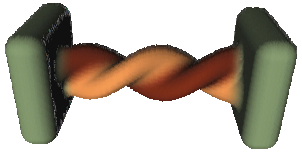
Deforming implicit primitives?

- F space deformation

  Ex: Scale, twist, bend, etc

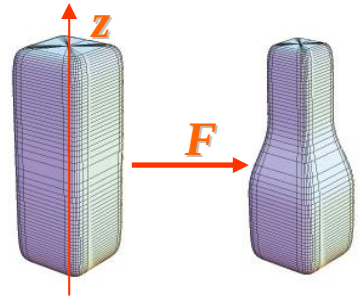


- Deformed implicit surface

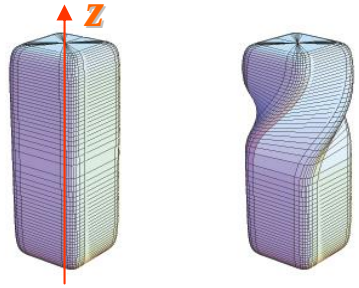


$$f_{deformed}(P) = f(F^{-1}(P))$$

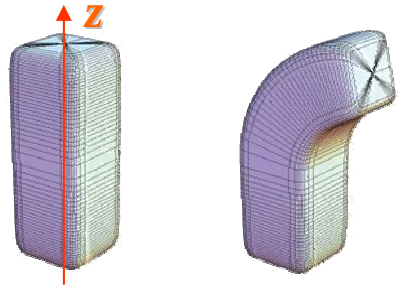
$$\begin{pmatrix} s(z) & 0 & 0 & 0 \\ 0 & s(z) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \\ 1 \end{pmatrix}$$



$$\begin{pmatrix} \cos\theta(z) & \sin\theta(z) & 0 & 0 \\ -\sin\theta(z) & \cos\theta(z) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \\ 1 \end{pmatrix}$$



$$\begin{pmatrix} \cos\theta(z) & 0 & -\sin\theta(z) & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta(z) & 0 & \cos\theta(z) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \\ 1 \end{pmatrix}$$

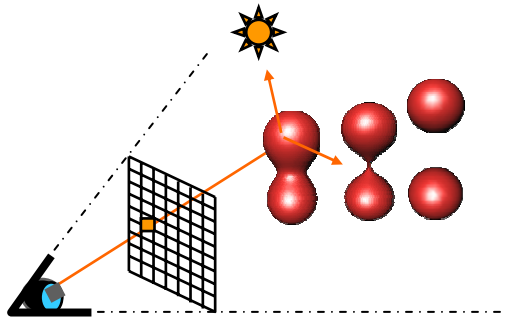


Example of use: Blob tree

- Inspired from CSG trees
  - Blending nodes (+, - , max, min, etc)
  - Unary deformation nodes
- Used for procedural modeling
  - Description file

Displaying implicit surfaces? Ray Tracing [Blinn 82]

- Use dichotomy to compute ray/surface intersections



Later extensions

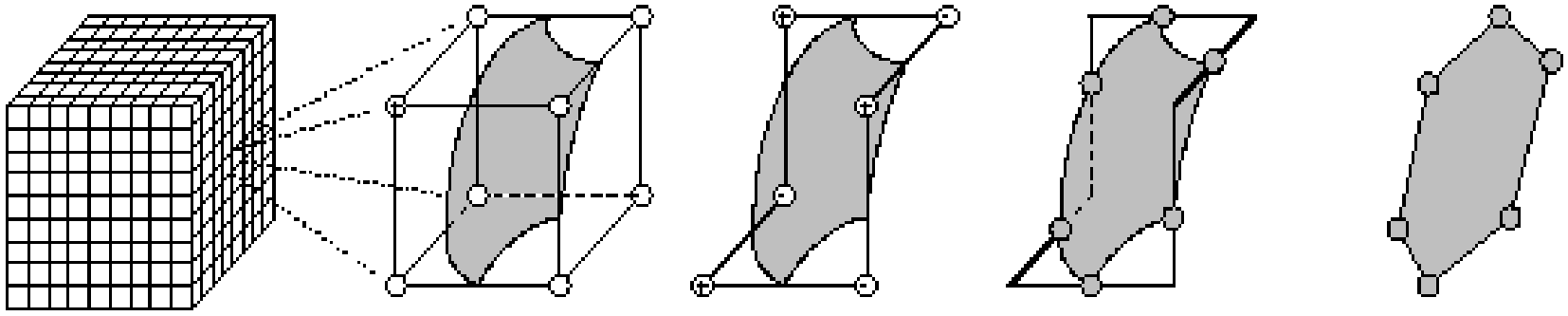
- Analytical solutions for intersection
- Sphere tracing
  - adapt the step size based on Lipzchisz constants

1980-2000: Several hours for rendering from a single view-point!

Converting implicit surfaces to meshes

Marching cubes *[Wyvill MP W 86, Lorensen Cline87]*

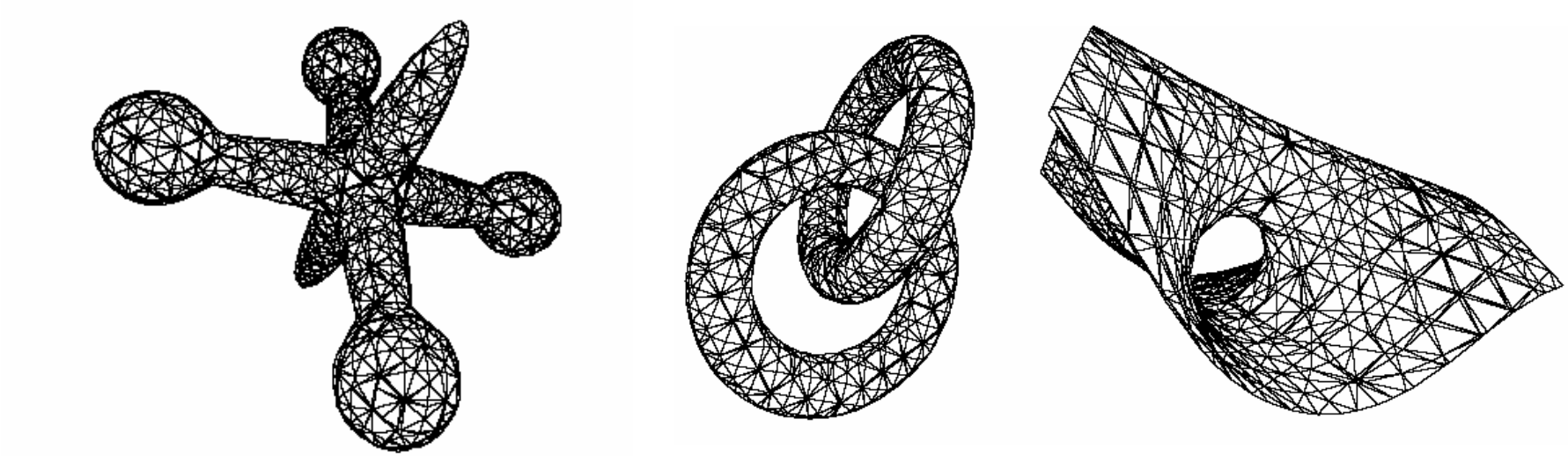
- Space grid
- Facetize voxels that cross the surface
- Mesh can be viewed from different viewpoints
- Extension: file to follow the surface



Converting implicit surfaces to meshes

Marching cubes *[Bloomenthal 1993-1994]*

- Evaluation of implicit surface tilers
- An implicit surface polygonizer (paper + code in C)

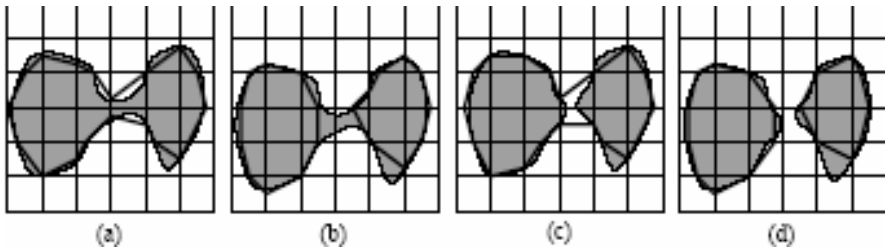


Advanced bibliography

Guaranteeing the Topology of an Implicit Surface Polygonization

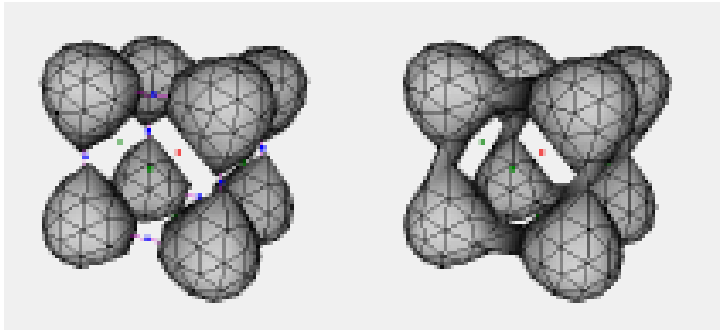
[Stander Hart SIGGRAPH 1997]

- Morse theory used to track critical points
- Garanteed correct topology!



Marching cube correct on (a) (d)

but fails on translated shapes (b) & (c)



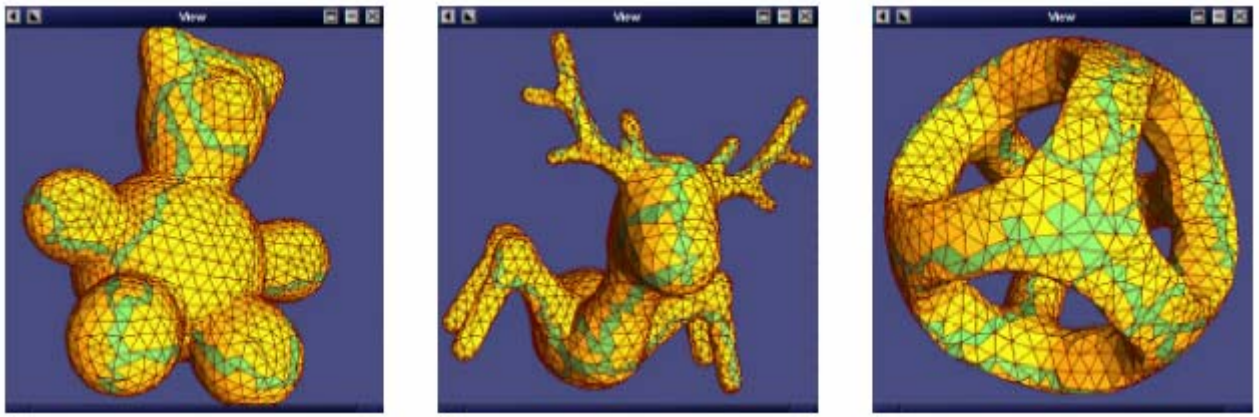
Tracking critical points

Advanced bibliography

Adaptive Implicit Surface Polygonization Marching Triangles

[Galin Akkouche, Computer Graphics Forum 2001]

- Good quality meshing of implicit surfaces
  - marching triangles, instead of marching cubes
  - Size adapted to local curvature
  - Use in an interactive modeling system

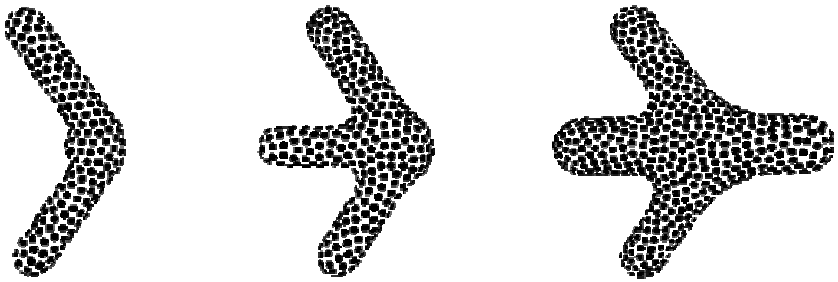


Interactive modeling with implicit surfaces?
Fast visualisation

Particules rendered as splats in the tangent plane

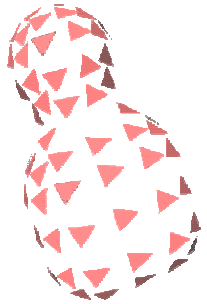
- *[Bloomenthal Wyvill 1991]*
  - Random particles projected along the field gradient

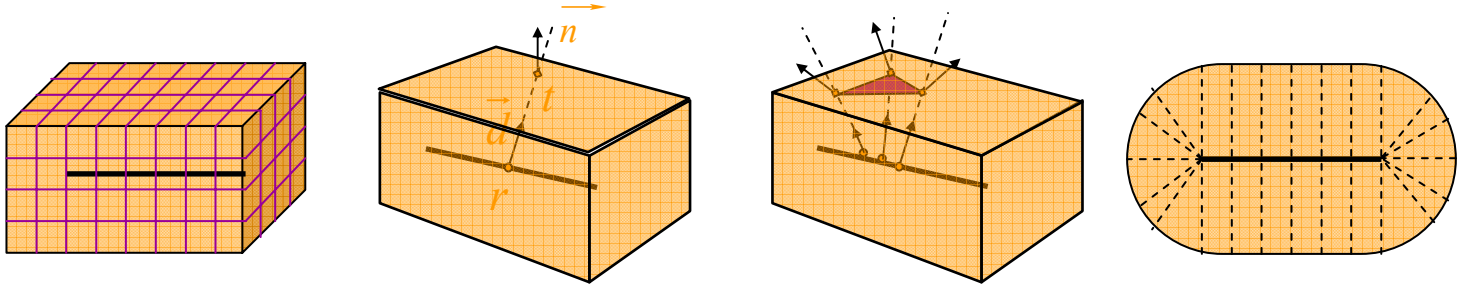
- *[Witking Heckbert 1994]*
  - Attraction/repulsion forces
  - Constrained to remain on the surface
  - Split/death of particles

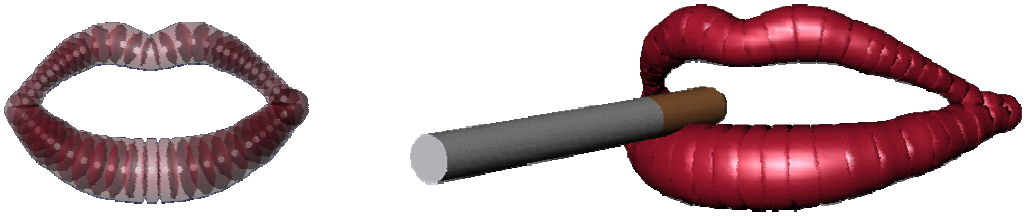


Interactive modeling with implicit surfaces?
Fast visualisation

- *[Desbrun, Tsingos, Cani 1995]*
  - Sampling of primitive ‘territories’
  - Piece-wise polygonization





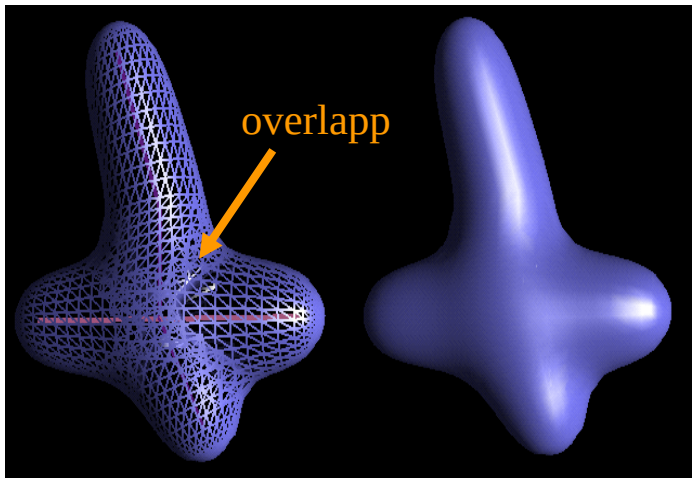


Fast visualization [Cani Hornus 2001]

- Overlapping territories

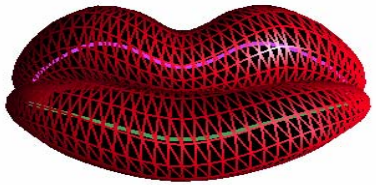
$$\left\{p \mid \forall j \neq i,\ f_i\ (p) + \eta > f_j(p)\right\}$$

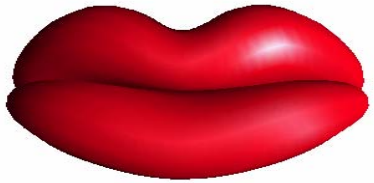
Real-time rendering using OpenGL



- A closed polygonal mesh for each skeleton curve

Old lips

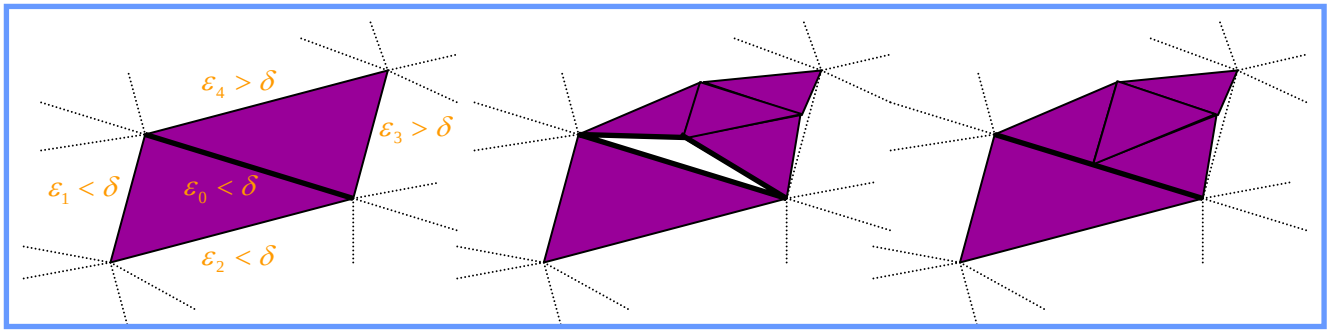


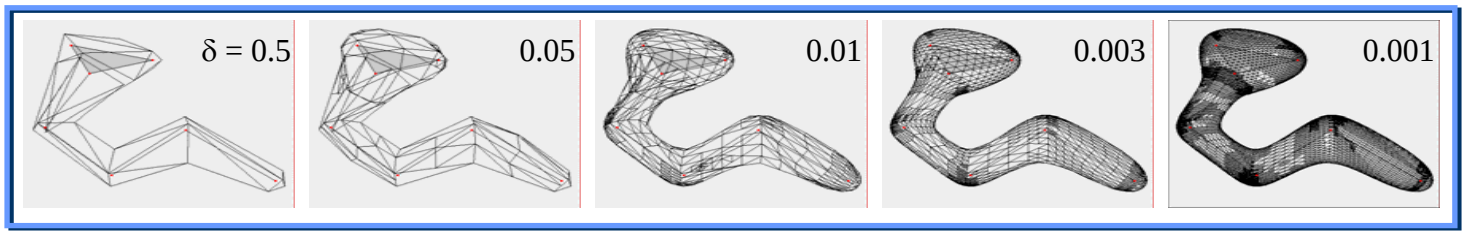


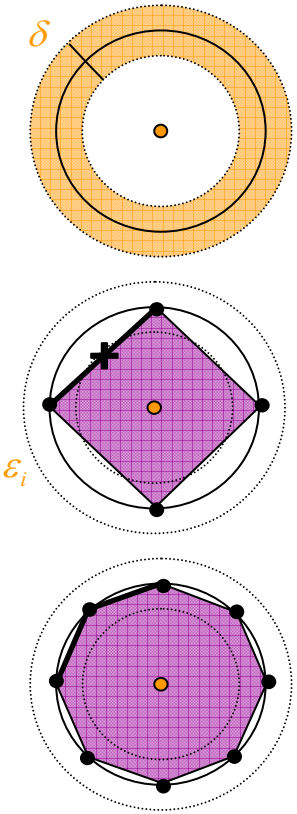
New lips

Fast visualization [Angelidis Cani 2002]

- Refinement criteria: field well reconstructed?
- Avoid cracks

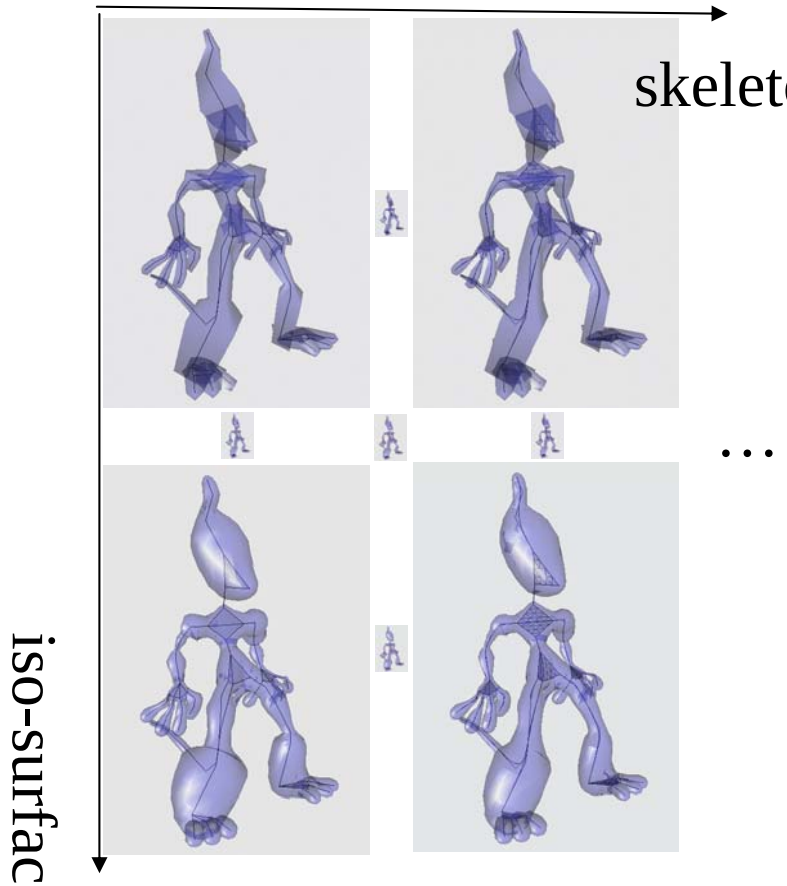






Subdivision curves & surfaces as skeletons

Results [Angelidis Cani 2002]



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